New ways of discretizing the atmosphere for numerical weather prediction

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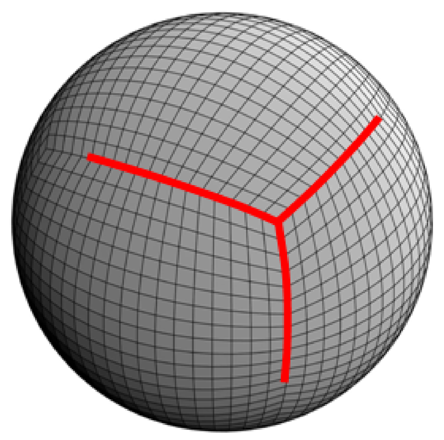
1. Introduction – Weather prediction is an agonizingly multi-faceted problem. Here we consider alternatives for discretizing the 3-D atmosphere and the differential equations governing its evolution so they can be accurately solved on a sphere. The traditional approach - discretization in latitude-longitude space - creates numerical singularities at the two poles. Alternatives proposed to alleviate the pole problem include the cubed sphere, the Yin-Yang grid, the Fibonacci grid, and the icosahedral grid.

2. The Cubed Sphere – A cube turned into a sphere by inflating it like a balloon. Popular because conventional *x,y* discretizations can be used on individual faces. The eight corners of the cube require special treatment.

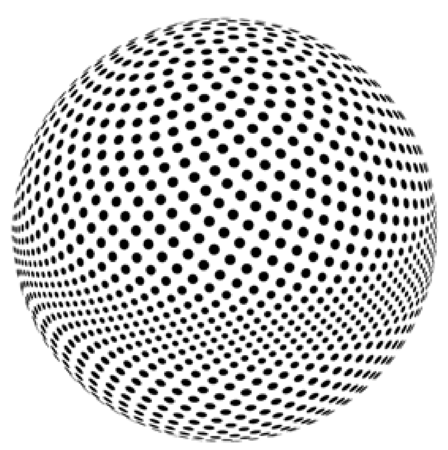


(Picture credit: J. W. Hernlund, P. J. Tackey, Comp. Fluid Solid Mech., 2003)

3. The Yin-Yang Grid – Two pieces resembling cupped hands pressed together to form an enclosure. Conventional *x,y* discretizations can be used, but information transfer between the Yin and the Yang grid requires overlaps and interpolation.

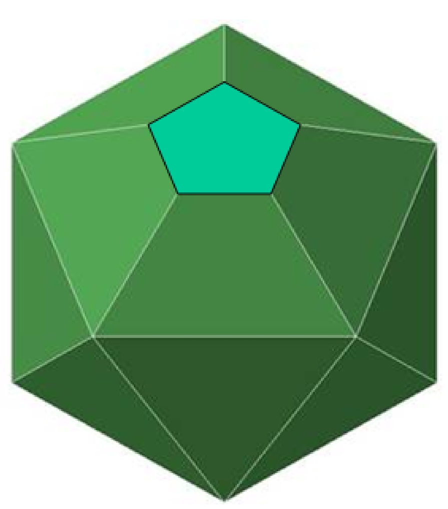
(Picture credit: Takahashi *et al*, Proc. 7th Int. Conf. HPCAsia'04)

4. The Fibonacci Grid – Grid point locations mimic the way nature distributes discrete objects in finite areas (such as seeds in seed pods).

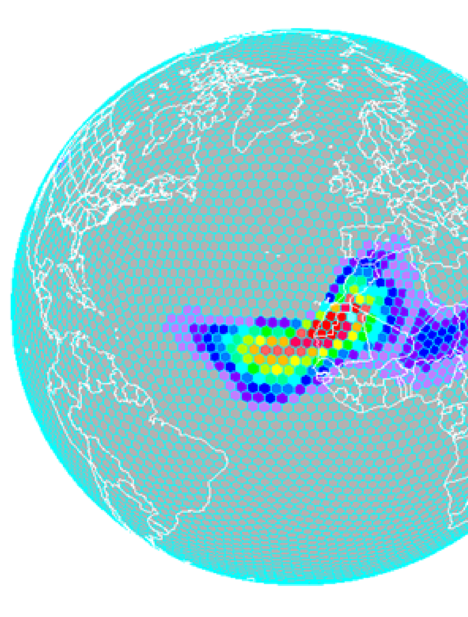


(Picture credit: Swinbank and Purser 2005)

5. The Icosahedral or "Soccer Ball" Grid – The black and white patches on a soccer ball are created by fitting white hexagons into each of the 20 triangles in an icosahedron and combining the leftover triangle fragments into 12 black pentagons.



By repeatedly subdividing triangles into four smaller ones and combining the final set of triangles into hexagons and pentagons, honeycomb-like high-resolution grids can be created. The number of pentagons remains constant during the refinement process, but the number of hexagons can be arbitrarily large.



(Picture credit: G.Grell, NOAA-ESRL)

The severe 2-pole problem in the traditional lat-lon grid is thereby "diluted" into 12 rather benign grid anomalies. Because of the near-circular shape of grid cells, this grid is ideally suited for the **finite volume** approach [7] where differential operators (divergence, vorticity, gradient) are expressed as line integrals along the perimeter of a grid cell.

One obstacle to using this grid is that traditional 2-D discretizations cannot be used. A weather prediction model using an icosahedral grid must be built from scratch.

(Related work: [8,9,10,12])

6. Introducing FIM, a "Finite-Volume, Flow-Following, Icosahedral" Weather Prediction Model – In FIM, recently developed at ESRL, the underlying equations are discretized on an icosahedral grid consisting of up to 655,000 cells (mesh size ~30 km). In a second break with convention, FIM features a nonstandard **vertical** discretization. Cloud and radiation processes are based on those used in the U.S. Weather Service's GFS model.

For more detailed documentation, see http://fim.noaa.gov.

6a. The Vertical Grid – Computer models cannot solve differential equations; they actually solve sets of algebraic equations. Substituting algebraic for differential equations gives rise to "dispersion" errors during horizontal and vertical transport. One can hide these errors behind a smoke screen of naturally occurring mixing/stirring processes by aligning coordinate surfaces with surfaces along which stirring preferentially occurs. These surfaces typically coincide with surfaces of constant entropy, suggesting that transport calculations be carried out in an isentropic coordinate system. FIM uses such a coordinate system, modified ("hybridized") to avoid intersections of coordinate surfaces with the ground.

Reducing **lateral dispersion** errors is only one aspect of an isentropic coordinate system. Since entropy is conserved during gravity wave-induced vertical motion, isentropic coordinate surfaces follow the ups and downs of wave motion. Hence, there is no vertical interlayer transport during the passage of gravity waves, i.e., no **vertical dispersion**.

The "flow-following" aspect of the coordinate requires spacing of layer interfaces to be time-dependent.

6b. Vertical Grid Hybridization – Coordinate surface-ground intersections are avoided in FIM by combining isentropic coordinate surfaces aloft with terrain-following (σ) surfaces near the ground. The algorithm managing interactions between the isentropic and the σ-coordinate subdomain is based on the ALE (Arbitrary Lagrangian-Eulerian) scheme [4].

Like ALE, the algorithm built into FIM maintains non-zero separation between coordinates surfaces by transferring mass between layers. This unfortunately makes layers lose their isentropic character. To counteract this trend, which is exacerbated by diabatic processes in the atmosphere, our algorithm continually checks for opportunities to restore isentropic conditions in a layer [1,3].

Restoration of "target" entropy (or its proxy, potential temperature θ) is accomplished by entraining air from a neighboring layer of different entropy. In determining the rate of transfer, maintenance of minimum layer thickness trumps "target" restoration.

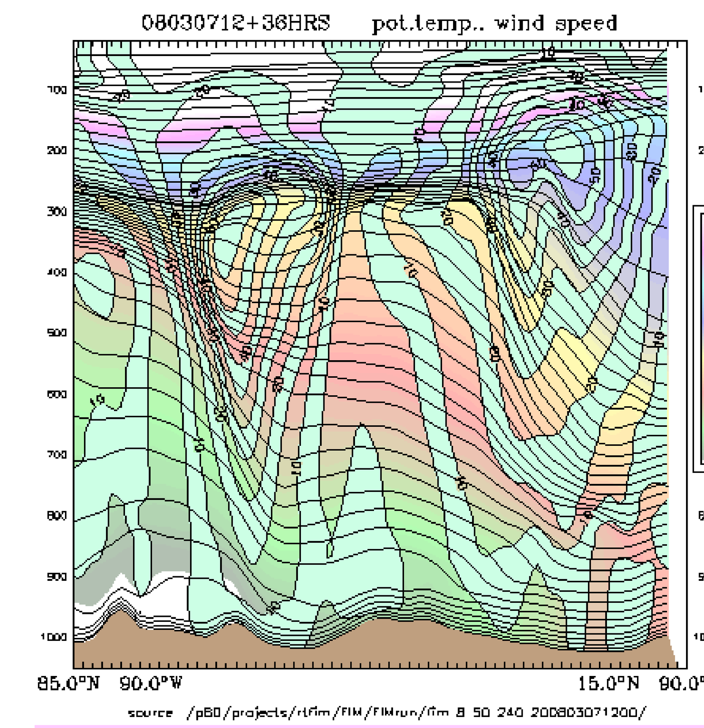
Vertical migration of grid points and interlayer mass transfer are simultaneously inferred from the mass conservation equation written in the form

$$\begin{pmatrix}\text{vertical}\\ \text{motion}\\ \textit{of}\\ \text{coordinate}\\ \text{surface}\end{pmatrix} + \begin{pmatrix}\text{vertical}\\ \text{motion}\\ \textit{through}\\ \text{coordinate}\\ \text{surface}\end{pmatrix} = \begin{pmatrix}\text{vertically}\\ \text{integrated}\\ \text{horizontal}\\ \text{mass flux}\\ \text{divergence}\end{pmatrix}$$

where only the right side is known initially. The ALE algorithm provides the extra condition needed to determine the two terms on the left [2]. Traditional hydrostatic models set first term on the left to zero.

(Related *non*-ALE hybrid coordinate work: [5,6,11,13].)

7. Drawbacks – No weather prediction model is superior to others in all possible respects. Known shortcomings of the θ coordinate include poor vertical resolution in unstratified (constant-θ) and thermally unstable air columns. Abrupt changes in vertical resolution can occur at the θ–σ interface. Time- and space-dependent layer spacing requires sophisticated transport schemes for conservation. At present, FIM is a hydrostatic model – a handicap in simulating buoyant convection and associated cloud processes.



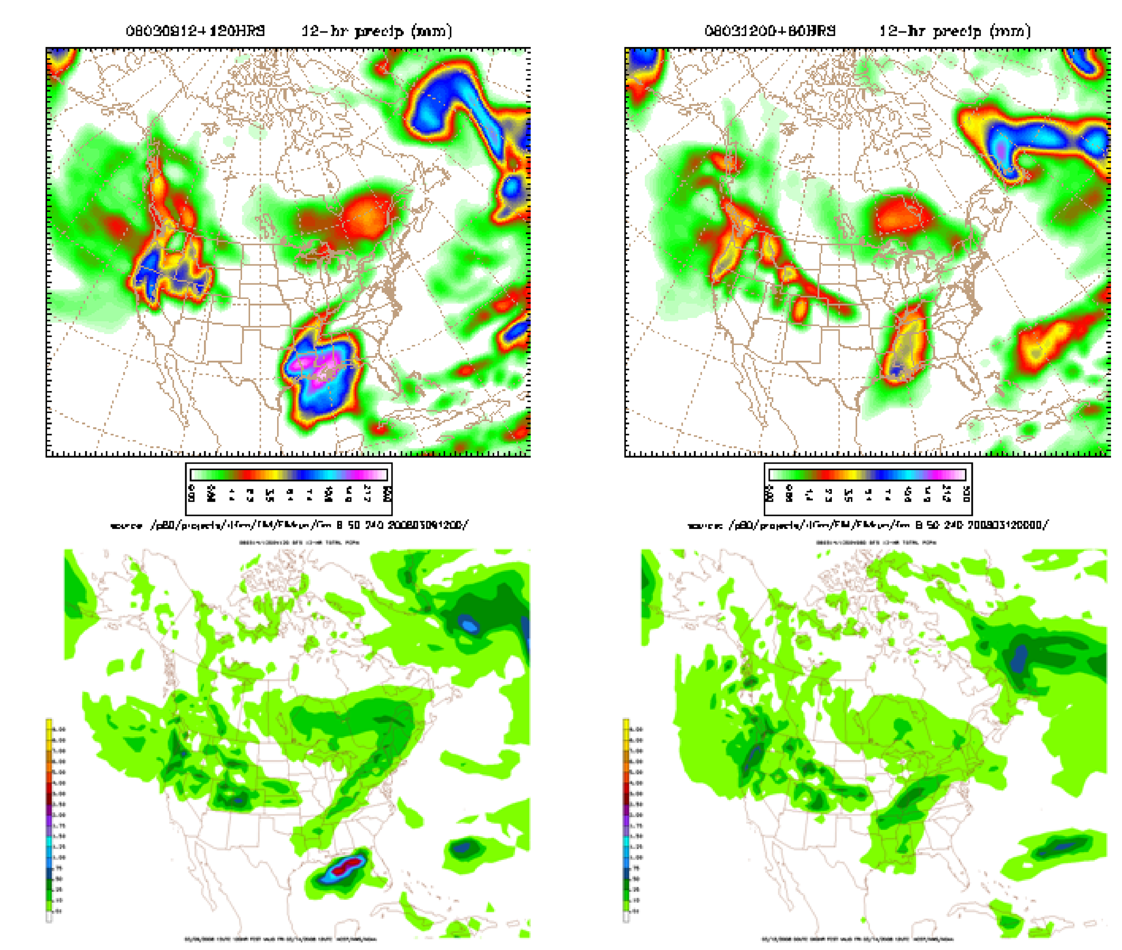
Meridional vertical section through the model atmosphere, showing coordinate surfaces (solid lines), wind speed (m/s, shaded contours), and θ (°K, color).

Ordinate: pressure (hPa)

The above figure illustrates aspects of FIM's vertical coordinate. In the free atmosphere, color follows coordinate layers, indicating that layers are isentropic. Near the ground, layers follow the terrain. Due to the north-south temperature contrast, the σ domain extends up higher in the south than in the north. This is not unwelcome as it provides "guaranteed" resolution for the simulation of convective processes which are more prevalent at low latitudes.

Two jet streams are shown: the polar one on the left and the subtropical one on the right. The packing of isentropes beneath both jets indicates the presence of upper-tropospheric fronts which represent extrusions of stratospheric air into the troposphere. Simulation of fronts is one of the strengths of the θ coordinate system.

8. A Rudimentary Case Study



5- and 2.5-day (left/right) precipitation forecasts for the 12-hr period ending 1200 GMT, 14 March 2008, from FIM and GFS (top/bottom). Caution: map projections, units, color bars differ.

Salient points: (1) The phase speed of rain-producing synoptic systems is remarkably close in the two models. (2) One noteworthy difference is in the predicted extent of precipitation over the central Gulf states. Both models stray from the "perfect" forecast – two distinct rainfall maxima, one offshore and one on the Arkansas-Missouri border. In this particular case, neither model outperforms the other.

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